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MATHEMATICAL MODELING OF THE HYDROECOLOGICAL PROCESSES IN WATER BASINS

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Abstract. This article discusses the distribution of polluting substances from environmental pollution sources. Based on proposed model, it is possible to calculate the density of the mixture and to determine its critical level. Particular importance is given to the study of the movement dynamics of pollutants that migrate from the source and the sources of origin to the water basins.

Keywords: dissemination, pollution, pollution sources, water environment, mathematical models, calculation of mixture concentration. water basin ecosystem.

AMS Subject Classification: 93A30, 97Mxx.

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1 Introduction

There are not enough simple models that can be applied in practical reports for the theoretical study of pollution of water, soil, atmospheric and other environments. It should be noted that the turbulent diffusion equations characterizing the dissemination of mixtures in the water environment correspond to the qualitative Navier-Stokes equation, their natural flows and so on. When adding factors, the system of the equations becomes even more complicated and almost useless for the practical use.

The development of computational methods and techniques and the application of modern computer technologies to the study of hydroecosystems open up new opportunities for researchers and organizations dealing with the rational use of natural environment for the sake of human well-being and responsible decision-makers. Therefore, it is increasingly becoming more and more urgent to establish expert information systems for operational monitoring, differential assessment of the current state of the water basin ecosystem, providing short and long-term forecasts, high-skilled guidance.

Protection of water basins from anthropogenic impacts on external influences, improvement of water quality control mechanism is one of the most important practical issues of modern times and requires immediate action.

Let's examine the modeling of water pollution as a link between the formalities of two interconnected environments. One of the enzymes is water-soluble oxygen-containing waste of water, and the other is water-fired waste.

2 Statement of the problem

The following three main ways of entering dangerous substances into surface water bodies are:

- falling from the surface of the atmosphere;
- irrigation receipts;
- mixes occurring during fertilizing and other types of agrotechnical measures.

In the solution of practical issues operative calculations and the following model are offered, which can be used in short-term forecasting.

Scattering into the water basin over the surface layer of the atmosphere will be solved by accepting the entire surface of the water surface. The receiving surface of the water surface may be fixed along its fixed basin, but it is usually time-varying. Depending on the irrigated area, the crops grown in the irrigated area, and the other phytomiortic conditions, the amount of pollutants in this way will also be partly cyclical, but will vary according to time. Therefore, given these factors, we can write on a single surface, for a single time interval:

$$Q_a(t) = A_0 \cdot f_a(t) \cdot H(t), \tag{1}$$

$$A_0 = \frac{C_0 \cdot S}{S + S_0}.$$

Here $f_a(t)$ is the amount of precipitation falling from the atmosphere for one day (m^3/day); C_0 is the uniform density of pollutants; S is the nominal area of the water surface; $S_0(t)$ is the splinters from the nominal surface;
For contaminated ingredients during irrigation

$$Q_S(t) = A_1 \cdot f_S(t) \cdot \delta(t - k_S), \tag{2}$$

$$A_1 = \frac{C_1 \cdot S_0}{S + S_0}.$$

Here $f_s(t)$ is the amount of water entering the basin during the irrigation period; k is the time of irrigation. $\delta(t - k_s) = \begin{cases} 0, & k(t) \neq t \\ 1\,, & k(t) = t \end{cases}$ is the delta function. C_1 is the amount of pollutants contained in the irrigation water and soil water filtration.

During other agro-technical measures, we can write for pollutants in water basins as follows

$$Q_g(t) = A_2 \cdot f_g(t) \cdot \delta(t - \tau_g), \tag{3}$$

$$A_2 = \frac{C_g \cdot S_0}{S + S_0}.$$

Here $f_g(t)$ is quantity of fertilizer per unit area; C_g is the density of hazardous substances in fertilizer; $\delta(t - t_g)$ $\delta(t - -\tau_g)$ is the delta function; δ_g is a time of fertilization.

$$\delta(t - \tau_g) = \begin{cases} 0\,, & t \neq \tau_g \\ 1\,, & t = \tau_g \end{cases}$$

The total amount of contaminants involved in the water basin can be calculated by formula

$$Q\,(t) \;= Q_a\,(t) \;+\; Q_s(t) \;+ Q_g\,(t).$$

This model is of a general nature, and many parameters require certain definitions for specific situations.

These parameters can include the following:

- physical and chemical properties of contaminants;
- plant, soil, atmosphere and so on. ecological-hydroerological properties of participating environments;

- landscape-ground characteristics of the area where the water basin is located;
- meteorological factors.

This model can be further enhanced in real processes through the use of advanced processing of informative materials and application of modern information technologies based on long-term measurements and observations.

The disintegration of a member into the water environment results in the effect of oxygen on the bacteria that generate a chain of chemical reactions. Therefore, interaction between oxygen and wastes in water is modeled.

3 Solution of the problem

Waste concentration is determined by the specific unit of measurement called oxygen biochemical demand (OBT). OBT is the amount of oxygen essential for waste disintegration to the volume of water (mq / l). Waste disintegration rate is proportional to their thickness

$$\frac{dC}{dt} = -k_0 \cdot C. \tag{4}$$

Here k_0 is oxygen attenuation constant and is usually expressed in (day) -1 units.

If C_0 does not have a waste water (pure water), the oxygen content of the oxygen, i.e. the known function dependent on temperature, will be $C, < C_0$. If we accept $D = C - C_0$, then

$$\frac{dD}{dt} = k_0 \cdot C - k_1 D. \tag{5}$$

Here $k_o C$ describes the process of oxidation of waste, $k_1 D$ is reaceration (the absorption of oxygen in surface water), k_1 is called the residual coefficient.

So we have a system consisting of two equations

$$\begin{cases} \frac{dC}{dt} = -k_0 \cdot C \\ \frac{dD}{dt} = k_0 \cdot C - k_1 D \end{cases}. \tag{6}$$

This system is simplistic in its mathematical terms and adequately reflects the real process. The solution of this system is as follows:

$$D(t) = \frac{k_1}{k_2 - k_1} C(0)(\exp(-k_0 t) - \exp(-k_1 t)) + D(0) \cdot \exp(-k_1 t),$$

$$C = C_0 - D.$$

Here $C(0)$ and $D(0)$ are the values of the corresponding function at the time $t = 0$. For the small water reservoirs $C(0) = C_0$ can be accepted.

There is a question that is very interesting in practice: What should be the maximum value of saturation with oxygen if waste of water is thrown into a certain part of the river or water basin? Because, when the oxygen indicator is lower than a certain value, irreversible processes begin in the water basin (fishes are destroyed and others) which can turn the water basin into a dead environment. To determine the maximal oxygen deficiency, find the derivative of the function $D(t)$ and solve the equation $D'(t) = 0$. Then,

$$D_{\max} = C(0) \cdot \frac{k_0}{k_1} \cdot \left(\frac{k_1}{k_2} \left(1 - \frac{D(0)}{C(0)} \cdot \frac{(k_1 - k_0)}{k_0} \right) \right)^{\frac{k_0}{k_0 - k_1}}.$$

Here $C(0)$ and $D(0)$ are the initial values of waste density and oxygen deficiency, t is associated with the x distance of the waste site. If the river flow velocity is V, then $x = Vt$. In this case $D(0)$ reflects the influence of pollutants (factory and others) located in the upper stream of the river.

To ensure an environmental standard, ie the environmental safety threshold in the aquatic environment, $D_{\max} < D_{\lim}$ must be provided, where $D_{\lim}$ is the limit determined as a result of appropriate research.

The movement of water in the basin and the dissemination of mixtures in the aquatic environment require the solution of a system of special derivative differential equations that expresses basic physical laws. Almost all regular continuous models are solved in various variants of Navier-Stokes equations that finding a precise analytical solution of these issues is related to great mathematical difficulties and is not yet possible. In these matters, the Navier-Stokes equations represent the "hydrodynamic" part of the problem, and the convective-diffusion equations should be added to them. Therefore, the study of these issues by numerical methods is of great relevance. Use of these models allows for abandonment of very costly natural observations and prolonged environmental forecasts.

Let's consider the action of a mixture of water and non-soluble contaminants that fill up a specific spatial area. V_0 particle particles have been included in the V volumes in the initial $t = 0$. These particles fall into the bottom of the water are very complicated, and they move along with the surrounding water environment and cause the mix to change in the local density. Under the influence of the gravity force and the mutual hydrodynamic impact forces, these particles move forward and move along with the liquid that covers them. We can take into account the effects of inertia between fluid and solid phases, because the particles' specific densities are very close to each other. In this case the local average speed of the mixture satisfies the following equation:

$$\frac{\partial\,(\overrightarrow{u}\rho)}{\partial\, t} + (\nabla\overrightarrow{u})(\overrightarrow{u}\rho) - 2\mu\sum_{i=1}^{3}\frac{\partial}{\partial\, x_i}(a_{ijkl}(x,t) - \varepsilon_{ij}(u))\cdot\overrightarrow{e}^{\,i} + \nabla P = \overrightarrow{f}\rho, \quad div\overrightarrow{u} = 0, \qquad (7)$$

where $\overrightarrow{u} = \overrightarrow{u}(x,t)$ is an average motion velocity of the mixture, $P = P(x,t)$ is a pressure,

$$\rho = \rho(x,t) = \rho_m(1 - C(x,t)) + \rho_b C(x,t)$$

is an average density of the mixture $C(x,t)$ is a medium particle density, μ is a liquid dynamic absolute coefficient, $\overrightarrow{f}$ is the main vector of external forces that affects the mixture; $\varepsilon_{ij}\cdot = \frac{1}{2}\left(\frac{\partial\, u_i}{\partial\, x_j} + \frac{\partial\, u_j}{\partial\, x_i}\right)$ is a deformation tincture of concordant liquid; $\overrightarrow{e}^{\,i}$ are the single base vectors, $a_{ijkl}(x,t)$ is a substance of self-esteem.

Calculation of this tensor is associated with great difficulties, so when the viscosity $C(x,t)$ is small, it can be expressed by asymptotic formulas. When applied to practical matters, it is caused by great complexity, it is centered on the isotropic tensor, ie the solid particle movements in the mixture are modeled as bridge flows and are identified by the effective absorption coefficient.

$$\mu_{ef} = \mu(1 + \gamma C(x,t)), \qquad (8)$$

where γ is the coefficient dependent on the current form of the particle.

In the general case, we can write the following equation because the particle tissue particles are dependent on the $\overrightarrow{\lambda}(x,t)$- orientation vector and $C(x,t)$.

$$\frac{\partial\, C(x,t)}{\partial\, t} + (\overrightarrow{u}\nabla)C = 0. \qquad (9)$$

This type of equivalence must also satisfy the density $\rho(x,t)$.

Thus, we can express the movement of a mixture of weakly dense (non-squeezed fluid) by the following system the of closed-loop equations:

$$\frac{\partial \overrightarrow{u}}{\partial t} + (\overrightarrow{u}\nabla)\overrightarrow{u} - \frac{1}{\rho}\sum_{i,k=1}^{3}\frac{\partial}{\partial x_i}(\mu_{ef}(\rho)\cdot\frac{\partial \overrightarrow{u}}{\partial x_i}) = \overrightarrow{f} - \frac{\nabla\rho}{\rho}, \quad div\overrightarrow{u} = 0,$$

$$\frac{\partial\rho}{\partial t} + (\overrightarrow{u}\nabla)\rho = 0, \tag{10}$$

$$\mu_{ef} = \mu\left(\frac{\rho_b - (1+\gamma)\rho_m}{\rho_b - \rho_m} + \frac{\gamma\rho(x,t)}{\rho_b - \rho_m}\right).$$

Let's assume that at the moment at $t = 0$ into the water basin of the volume Ω the waste of volume $\Omega_0 \subset \Omega$ was injected. When the particles are initially distributed, the density can be as follows:

$$\rho\,(x,\,0) = \begin{cases} f_1\,(x,\,0),\ x \in \Omega_0, \\ \rho_m,\ x \in \Omega\setminus\Omega_0. \end{cases} \tag{11}$$

The initial rate of contamination of the contaminants, i.e. the initial condition of the mathematical problem is $u\,(x,\,0) = f_2\,(x)$. There is only one (vertical) fixed component of the arc volumes ρg ρg, $g = 9.8m/s^2$ is a stabilizing momentum.

Given the non-linearity and complex mathematical nature of the problem, we will use the ANSYS software complex (Vorovich, 1981). The issue of physical properties for the successful application of this complex in the solution of the problem is divided into two parts. In the first case, the velocity and pressure is calculated by taking into account the flow, and the second is the dynamics of pollutants spread in the already known speeds. Reports use ANSYS / FLOTRAN and TERMAL (Goncharova, 2002) software modules based on the finite-element method. The results of reporting for conventional water holders are given in the section "Supplements" of the dissertation. In addition, the problem for the rectangular regions was found to be well compromised with the results obtained using two ANSYS / FLOTRAN and Flex Pde software packages. As a result, they offer a newer method of research on classroom issues. That is, the speeds are determined via ANSYS / FLOTRAN, the results are imported into Flex Pde and the problem of transporting and disseminating the mixture is solved.

One of the main reasons for contamination of water basins is the discharge of waste from industrial, communal and agricultural enterprisers to those who fail to pass through the initial cleaning systems. In this case, machine-information models have been used extensively for the study of mechanical and physical-chemical processes in the water holders (Vorovich, 1981), (Rozanov, 1995), (Sattarov, 2007). Using this type of perfect model will allow you to thoroughly analyze all the shades of the process, evaluate the effectiveness of different methods, and develop optimal process-targeted methods. Using this perspective and progressive technology requires adequate and extensive application of mathematical models that are practically possible in the physical process. In this case, model imaging models should consist of automotive models that model the process and provide pseudophysics imitation.

4 Results

Most of the processes taking place in the water holders are the study of the dynamics of change in the density of pollutants. This dynamics has a great impact on the state of the water-ecological system. Establishing the mathematical model of the process requires the removal of the moving equations representing the changing characteristics of the water quality characteristics and the solution of mathematical boundary issues. The reason why these issues are mathematically complex is the complex and multifaceted nature of the investigated processes, but also the fact that many of the parameters are random and fuzzy. In polluting mixtures in large water basins with multiple input-outputs, breakdown of member-constituents and so on. In the processes, the turbulent flow of water and internal streams play a very important role. Therefore, when creating mathematical models characterizing water quality in water basins, the factor of turbulent diffusion must be taken into account. Various types of contaminants entering the water environment themselves are exposed to various influences. So, some of the solid particles fall into the bottom of the basin, and some of them migrate from the basin, for example, through watercourses, irrigation ditches and others. Member-contaminants enter the water basin into a chemical oxidation reaction with free oxygen soluble in water, forming feed bases for living organisms in the basin, turning into non-member components and other water-soluble chemical compounds. Combustion of mercury, phenol, and other pollutants that have a destructive effect on the water basin ecosystem can cause a catastrophic catastrophe and can lead to the destruction of the basin's biodiversity.

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